

Question-1

Maximize $z = 5x_1 + 3x_2$

Subject to $2x_1 + 5x_2 \leq 50$

$x_1 + 3x_2 \leq 25$

$4x_1 + x_2 \leq 18$

$x_1 + x_2 \leq 12$

$x_1, x_2 \geq 0$

Suppose s_1, s_2, s_3 and s_4 are slack variables. Then

Maximize $z = 5x_1 + 3x_2$

Subject to $2x_1 + 5x_2 + s_1 = 50$

$x_1 + 3x_2 + s_3 = 25$

$4x_1 + x_2 + s_2 = 18$

$x_1 + x_2 + s_4 = 12$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Then first simplex tableau

y_1	y_2	s_1	s_2	s_3	s_4	z	
2	5	1	0	0	0	50	
1	3	0	1	0	0	25	
4	1	0	0	1	0	18	
1	1	0	0	0	1	12	
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-5	-3	0	0	0	0	1	0

Question-2

$$\text{Maximize } z = 5x_1 + 8x_2 + 6x_3$$

$$\text{Subject to } x_1 + x_2 + x_3 \leq 90$$

$$2x_1 + 5x_2 + x_3 \leq 120$$

$$x_1 + 3x_2 \geq 80$$

$$x_1, x_2, x_3 \geq 0$$

Suppose s_1 and s_2 are slack variables and s_3 is surplus variable. Then

$$\text{Maximize } z = 5x_1 + 8x_2 + 6x_3$$

$$\text{Subject to } x_1 + x_2 + x_3 + s_1 = 90$$

$$2x_1 + 5x_2 + x_3 + s_2 = 120$$

$$x_1 + 3x_2 - s_3 = 80$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Then first simplex tableau

Y1 y2 y3 s2 s3 z

$$1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 90$$

$$2 \quad 5 \quad 3 \quad 1 \quad 0 \quad 0 \quad 120$$

$$1 \quad 3 \quad 0 \quad 0 \quad -1 \quad 0 \quad 80$$

$$5 \quad 3 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0$$

Question-3

$$\text{maximize } z = 30x_1 + 40x_2$$

subject to

$$x_1 + x_2 = 60;$$

$$5x_1 + 10x_2 \leq 350;$$

$$10x_1 + 20x_2 \geq 400$$

$$x_1, x_2 \geq 0$$

Now using slack and surplus

$$\text{maximize } z = 30x_1 + 40x_2$$

subject to

$$x_1 + x_2 = 60;$$

$$5x_1 + 10x_2 + s_1 = 350;$$

$$10x_1 + 20x_2 - s_2 = 400$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Tableau #1

x1	x2	s1	s2	z	
5	10	1	0	0	120
10	15	0	-1	0	200
-20	-30	0	0	1	0

Tableau #2

x1	x2	s1	s2	z	
1/2	1	1/10	0	0	12
5/2	0	-3/2	-1	0	20
-5	0	3	0	1	360

Tableau #3

x1	x2	s1	s2	z	
0	1	2/5	1/5	0	8
1	0	-3/5	-2/5	0	8

Optimal Solution: $z = 480$; $x_1 = 24$, $x_2 = 0$

Question-4.

(a) Problem is of maximization having objective function

Max $Z = 6x_1 + 7x_2 + 5x_3$ and having all constraint " \leq " signs.

Subject to

$$4x_1 + 2x_2 + 3x_3 \leq 9$$

$$5x_1 + 4x_2 + x_3 \leq 10$$

And $x_1, x_2, x_3 \geq 0$

(b) If 1 is replaced by -1 then 1st constraint change to " \geq " signs

(c) according to final tableau, $x_1 = 0$, $5x_3 = 8$, $10x_3 = 21$

Thus

$$x_1 = 0, x_2 = 1.6, x_3 = 2.1.$$

(e)

Dual

$$\text{Min } Z' = 9w_1 + 10w_2$$

Subject to

$$4w_1 + 5w_2 \geq 6$$

$$2w_1 + 4w_2 \geq 7$$

$$3w_1 + w_2 \geq 5$$

Solution of this problem is remain same due to its primal problem having optimal solution.

Thus optimal solution is $w_1 = 1.3$, $w_2 = 2.1$ and optimal $z' = 22.7$

Question-5

Production Roberta Hernandez sells three items, A, B, and C, in her gift shop. Each unit of A costs her \$5 to buy, \$1 to sell, and \$2 to deliver. For each unit of B, the costs are \$3, \$2, and \$1, respectively, and for each unit of C, the costs are \$6, \$2, and \$5, respectively. The profit on A is \$4; on B, \$3; and on C, \$3. How many of each should she order to maximize her profit if she can spend \$1200 on buying costs, \$800 on selling costs, and \$500 on delivery costs?

Solution

Suppose Roberta Hernandez sell x_1 quantity of A, x_2 quantity of B and x_3 quantity of C. then according to his profit objective function is

$$\text{Max } Z = 4x_1 + 3x_2 + 3x_3$$

Each unit of A costs her \$5 to buy, \$1 to sell, and \$2 to deliver. For each unit of B, the costs are \$3, \$2, and \$1, respectively, and for each unit of C, the costs are \$6, \$2, and \$5, respectively. The profit on A is \$4; on B, \$3; and on C, \$3.

Then constraint is

$$x_1 + 3x_2 + 6x_3 \leq 1200$$

$$2x_1 + 2x_2 + 2x_3 \leq 800$$

And

$$2x_1 + x_2 + 5x_3 \leq 600$$

And all quantity can not be negative.

Then $x_1, x_2, x_3 \geq 0$

Question-6

The Aged Wood Winery makes two white wines, Fruity and Crystal, from two kinds of grapes and sugar. One gal of Fruity wine requires 2 bushels of Grape A, 2 bushels of Grape B, 2 lb of sugar, and produces a profit of \$12. One gal of Crystal wine requires 1 bushel of Grape A, 3 bushels of Grape B, 1 lb of sugar, and produces a profit of \$15. The winery has available 110 bushels of grape A, 125 bushels of grape B, and 90 lb of sugar. How much of each wine should be made to maximize profit?

Solution

Suppose Aged Wood winery produces x_1 quantity of Fruity and x_2 quantity of Crystal
Since Fruity and Crystal gives \$ 12 and \$15 profit respectively.

Then objective function is

$$\text{Max } z = 12x_1 + 15x_2$$

Now One gal of Fruity wine requires 2 bushels of Grape A, 2 bushels of Grape B, 2 lb of sugar, and One gal of Crystal wine requires 1 bushel of Grape A, 3 bushels of Grape B, 1 lb of sugar. Also The winery has available 110 bushels of grape A, 125 bushels of grape B, and 90 lb of sugar.

Then constraint are

$$2x_1 + x_2 \leq 110$$

$$2x_1 + 3x_2 \leq 125$$

$$2x_1 + x_2 \leq 90$$

$$x_1, x_2 \geq 0$$